Original Observations-based Control Charts for Monitoring the Mean of Auto-correlated Processes: A Comparison among Modified Shewhart, Modified EWMA, and ARMAST charts

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Abstract. Conventional control charts are considered satisfied enough to monitor the observations that are assumed to be independent and identically distributed (IID). Nevertheless, in real industry environment, the process data exhibits some serial dependence or autocorrelation in which the IID assumption is violated. With the presence of autocorrelation, the control limits of the control charts should be loosened since the tight control limits can decrease the in-control average run length (ARL); thus, lead to a higher false alarm rate. This paper aims to compare the performance and investigate the relative effectiveness of three control charts: modified Shewhart (mShewhart), modified EWMA (mEWMA), and ARMAST charts, based on the original observations when the process data follows ARMA(1,1) model. The parameters of those charts are manipulated to give the in-control ARL of 370. The simulation results shows that the mShewhart chart is not completely robust to the deviation from IID assumption for small mean shifts. Although the mEWMA chart is very good at detecting small mean shift, the optimal ARMAST chart outperforms the mEWMA chart when there is autocorrelation in the process. In addition, the ARMAST chart also surpasses the mShewart in monitoring large mean shift.

INTRODUCTION

A standard assumption that is largely cited in justifying the use of traditional control chart is that observations taken over time from the process under investigation are independent and identically distributed (IID). However, this assumption is often violated due to the presence of autocorrelation or serial dependence that possibly be the result of the dynamics, which is inherent to the process [1]. Autocorrelation might be occurred both in process industries and in parts industries. In process industries, such as production of foods and beverages, pharmaceuticals, chemicals, and petroleum, autocorrelation has long been recognized as a natural phenomenon, where parameters such as temperature and pressure differ slowly relative to the rate at which they are measured. On the other hand, only in recent years autocorrelation has become an issue in parts industries since processes are sampled more often and it is likely to detect autocorrelation that was previously undetected.

This serial correlation may seriously affect the performance of traditional control charts that are usually developed under the IID assumption [2]. Several researches have been conducted to investigate the effect of autocorrelation on the performance of traditional control charts. Reference [3] and [4] confirmed that inaccurate conclusions could be drawn by using conventional cumulative cum (CUSUM) chart in the presence of data correlation. Next, reference [5] showed that the average and median run lengths of CUSUM and exponentially weighted moving average (EWMA) charts were sensitive when the IID assumption was violated. Reference [6] revealed that in the presence of moderate levels of autocorrelation, an out of control point does not necessarily indicate a process change. On the performance of Shewhart control chart, the auto correlated could influence the false alarm rate [7]. Reference [8] proved

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theoretically that the run length of the autocorrelated process is larger than in the case of independent variables if all the autocovariance are greater than or equal to zero. Reference [9] found out that not diagnosing and considering the correlation in the data leads to a decrease in the average time to signal as the amount of correlation increases. Resuming the aforementioned studies, a typical effect of the presence of autocorrelation is that it produces bias estimators of the process standard deviation, and it would make the control limits tighter than expected. The tight control limits can cause to decrease the in-control average run length (ARL) leading to a higher false alarm rate. In addition, it can increase the time required to detect changes in the process. Therefore, one should not ignore the presence of serial correlation when designing the control charts.

There are two general approaches in constructing control charts to monitor autocorrelated process. The first which is proposed by [2], is to fit a time series model to the auto correlated data so that residuals or forecast errors form this model can be estimated. Assuming it is a true model, the residuals will be statistically independent; thus, the standard assumption is met and any traditional control charts can be applied. However, the residual charts do not have the same properties as the traditional charts and may have poor capability to detect the process mean shift [10]. The residuals chart also did not perform very well when the processes were positively autocorrelated at the first lag [5], [11]. In practice, the estimation process of the model parameters is difficult since the appropriate model to be used may not be clear.

In this paper, it is used another procedure, i.e. applying the original observations rather than the residuals to the control charts. It is necessary to adjust both control limits and the techniques for estimating process parameters to account for the autocorrelation. A good example is $\bar{x}$ charts to monitor autocorrelated data by [12]. Another approach is by [13] which proposed a procedure by plotting one-step-ahead EWMA predictions errors on a control chart.

The objective of this research is to compare and determine the relative effectiveness of the three control charts: modified Shewhart, modified EWMA, and ARMAST charts to monitor the mean of autocorrelated processes. The ARMA(1,1) model is used due to its characteristics: stationary, as many statistical process control (SPC) systems are in practice. It also contains both an autoregressive and a moving average component; hence, the effect of each type of parameter could be examined. Such attempt has been conducted by [14] that compared four control charts: Shewhart, EWMA, special-cause control (SCC), and common-cause control (CCC) charts. This paper did not consider the SCC and CCC charts since the second approach is employed. (The SCC chart is merely a Shewhart chart, but rather than plotting the original observations, the residuals are plotted, which are obtained after fitting the process with a time series model [2]; and the CCC chart is a chart of forecasted values that are determined by fitting the correlated process with a time series model [2].) To contribute, this paper considered ARMAST chart as a new SPC monitoring method. In fact, the ARMAST chart has been regarded to outperform SCC chart in the presence of autocorrelation [15].

PROCESS MODEL AND THE CONTROL CHARTS

ARMA(1,1) Process

Autocorrelated process can be captured using time series models. An important class of time series models are the stationary processes, which assume that the process remains in steadiness around a constant mean. If the observation at time t is referred as X_t, then the first order of autoregressive moving average process ARMA(1,1) is described as follows [16]:

$$X_t = (1-\phi)\mu + \phi X_{t-1} + a_t - \theta a_{t-1}, \tag{1}$$

where ϕ is the autoregressive parameter, θ is the moving average parameter, μ is the mean of the process, a_t is the random noise term at time t, assumed to be IID with mean 0 and variance of $\sigma^2{}_a$. When the process is uncorrelated, the values of ϕ and θ are 0. The process is stationary if $-1 < \phi < 1$ and invertible if $-1 < \theta < 1$. If $\phi = 0$ the process is said to be purely moving average (commonly written as MA(1)), and if $\theta = 0$ then the process is purely autoregressive or AR(1). The variance ($Var(X_t)$) of X_t as given by [17] are:

$$Var(X_t) = \sigma^2 x = \frac{1-2\phi\theta+\theta^2}{1-\phi^2}\sigma^2_{a_t}. \tag{2}$$

Modified Shewhart Chart

A modified Shewhart (mShewhart) chart is merely a Shewhart chart originally proposed by [18] in which the limits are adjusted to the case of autocorrelation. When the IID assumption is violated, the variance of the process is "estimated" badly. Therefore, the variance in (2) is used rather than the variance of the random noise. The process is considered to be out-of-control whenever the charting statistics falls outside the control limits of $\mu_X \pm L_X\sigma_X$, where μ_X and σ_X are the mean and standard deviation of the process; and L_X is a constant, which is selected to maintain the in-control ARL to the desired value.

Modified EWMA Chart

The EWMA chart by [19] is designed to detect small mean shift more quickly than the Shewhart chart. The charting statistics which are plotted are not the observations, but a "forecast" of a weighted sum of the current observation and the previous periods' forecast. If the forecast at time t is referred as H_t, then the charting statistics can be written as [20]:

$$H_t = \lambda X_t + (1-\lambda) H_{t-1}, \tag{3}$$

where λ is a smoothing constant which determines the weight given to past observations. When λ is large, relatively little weight is given to older observation, vice versa. Note that when $\lambda = 1$, the EWMA chart is Shewhart chart. Out-of-control signal is triggered when H_t falls outside the control limits, that is $\mu \pm L_H\sigma_H$, where σ_H is standard deviation of charting statistics, given as:

$$\sigma_H = \sigma_X \left(\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2t}\right]} \right). \tag{4}$$

The term $[1 - (1 - \lambda)^{2t}]$ in (4) approaches unity as t gets larger, thus the control limits will approach steady-state values:

$$\mu \pm L_H \sigma_X \sqrt{\frac{\lambda}{2-\lambda}}\ . \tag{5}$$

Analogue with the mShewhart chart, the main idea behind the modified EWMA (mEWMA) chart when the process is autocorrelated is that the charting statistics is compared with the standard deviation of the process; thus, the value of σ_X in (4) and (5) follow the square root of (2).

ARMAST Chart

ARMAST chart by [15] is a new charting technique based on the ARMA statistic applied to stationary processes. The ARMA statistic Z_t can be represented by:

$$Z_t = v_0 X_t - v X_{t-1} + u Z_{t-1}, \tag{6}$$

where u, v, and v_0 are charting parameters with $v_0 = 1 + v - u$ for the simplicity of steady state condition. X_t is a stationary process follows (1) that is constrained by $|v/v_0| < 1$ and $|u| < 1$. The constraints are necessitated for designing the ARMAST chart to guarantee that Z_t is reversible and stationary. It is easy to show that Z_t follows ARMA(2,2) process when ARMAST chart is applied to ARMA(1,1) model [15].

The control limit for ARMAST chart is given by $L_Z\sigma_Z$, where L_Z is a positive real value and σ_Z is a steady state standard deviation of the charting statistics. Therefore, the process is considered to be out-of-control whenever Z_t falls outside the control limits. The steady state variance of charting statistics is given as follows [21]:

$$\sigma_Z^2 = \left[v_0^2 + \frac{\sigma^2}{1-u^2} + 2\left(v_0\alpha + \frac{u\alpha^2}{1-u^2} \right)\frac{\rho_X(1)}{1-u\phi} \right]\sigma^2 x\,, \tag{7}$$

where $\alpha = uv - v$ and ρ_X (1) is first lag correlation coefficient given by: $\phi - \theta\sigma^2{}_a/\sigma^2{}_X$. The charting parameters and constant L_Z could be chosen to achieve a certain in-control ARL.

RESEARCH DESIGN

In order to attain a comprehensive view of the effect of the autocorrelation, this research was designed over the entire stationary region of ARMA(1,1) model. A five-level full factorial design was used in which values for the two model parameters (ϕ and θ) were chosen in the interval of (–1, 1) with the aim of the process to be stationary [22]. The parameters must not have equal values or the variance of the process would follow a random noise, see (2). The values of the model parameters are selected as follows: (i) strong autocorrelation: $\phi = \pm\ 0.95$; (ii) weak autocorrelation: $\phi = \pm\ 0.45$; (iii) strong moving average: $\theta = \pm\ 0.90$; (iv) weak moving average: $\theta = \pm\ 0.40$; and (v) no autocorrelation or no moving average $\phi = \theta = 0$.

A performance measurement for evaluating the aforementioned control charts used in this paper is the ARL, as is often utilized in standard SPC procedure, see for example: [11], [14], [15], [21], [23]–[25]. The ARL is defined as the average number of points or observations that must be plotted before a point indicates an out-of-control condition [1]. The ARL until an alarm is triggered when there is no mean shift in the process is denoted as ARL_0 or in-control ARL. Contrarily, out-of-control ARL or ARL_1 is the ARL until detection of a true mean shift. For a given control charts, it is desired to maintain the value of ARL_0 to be large when there is no mean shift and small value of ARL_1 when there is a mean shift in the process.

In this paper, the ARL_0 is maintained at value 370. The rationale behind the use of 370 of the in-control ARL is that it gives good results in practice. Longer in-control ARL will result in fewer investigations, but perhaps fewer process shifts will be promptly identified; while shorter in-control ARL will result in more investigations for assignable causes, and perhaps more false alarms. As a result, for evaluating the control charts' performance, the control charts with the lowest value of ARL_1 when the mean shift is occurred is considered as superior. This is analogous to matching the type I errors (probability of an out-of-control signal given no shift has occurred) so that the type II errors (probability of an in-control observation given a shift of a specific size has occurred) can be compared in a more meaningful way.

Consequently, control charts' parameters (mEWMA: λ, ARMAST: u and v) and constants (mShewhart: L_X, mEWMA: L_H, ARMAST: L_Z) were manipulated so that the ARL_0 is retained at value of 370. For mShewhart charts, when there is no autocorrelation in the process, the value of $L_X = 3$ is recommended to maintain the ARL_0 of 370; while there is serial correlation in the process, the values of L_X vary depend on the values of ϕ and θ, see [26], [27]. In the case of AR(1) model, smaller value of L_X has to be assigned for stronger autocorrelated process (bigger value of $|\phi|$), vice versa [26]. For example, when the coefficient of autocorrelation is 0.95 ($\phi = 0.95$, $\theta = 0$), the L_X is maintained at 2.491 to give the ARL_0 of 370. In the case of mEWMA charts, the values of λ and L_H vary depend on the parameters of the process. (When the observation follows IID assumption, the procedure by [23] could be used for various values of λ.) In this study, we fixed the value of λ in the number of 0.2 [28] while the values of L_H were manipulated so that the effect of L_H could be examined. The in-control ARL is maintained at 370. Lastly, the parameters u and v play important roles in the performance of the ARMAST chart. Transient and steady state signal-to-noise ratios (R_T and R_S) are used for choosing appropriate parameters of ARMAST chart. When the underlying process follows ARMA(1,1), R_T and R_S are defined as $R_T = v_0\mu/\sigma_Z$ and $R_S = \mu/\sigma_Z$. In this paper, it is used a heuristic algorithm developed by [15] to determine the appropriate values of u and v to keep the in-control ARL at the desired value.

The computation of ARL was determined via simulation. It is assumed that only one observation is available at each period and all parameters are assumed to be known exactly. The random noise is assumed to be normally and independently distributed (NID) with the mean of 0 and variance of 1 ($a_t \sim$ NID $(0,1^2)$). The observations were generated according to (1) with the various parameters of ARMA(1,1) model aforesaid. The process was repeated 250,000 times with the aim of obtaining the ARL. The standard error used in this study is ±1.96. It is convenient when monitoring the process mean to measure the size of the shifts in units of standard deviation of the process [28]. The size of mean shifts ranged from 0, 0.5, 1, 2, to 3 standard deviations of the processes. Note that there is no mean shift when the value is 0.

Some authors use MATLAB in estimating the ARL via simulation. However, we developed a Java-based application since it gives faster result when dealing with repeating iteration. The computational time for 250,000 iterations when using Java platform application is about 10 seconds compared with more than an hour when using MATLAB. The interface is shown in Fig. 1. The application is available from the authors on request. The algorithms are described below.

Algorithm mShewhart Chart Simulation

1: Determine the values of ϕ, θ, and L_X
2: Calculate σ_X using the square root of (2)
3: Calculate the control limits
4: Generate random number $a_t \sim \mathrm{NID}(0,1^2)$
5: Calculate X_t using (1)
6: **If** X_t falls within the control limits **then**
7:     **Repeat** 4–5
8: **Else**
9:   Calculate the Run Length (RL)

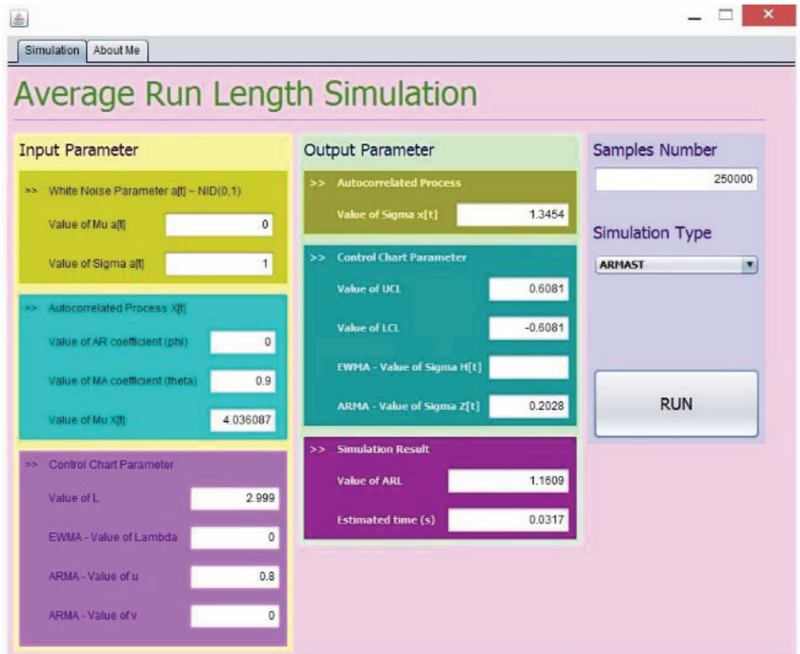

FIGURE 1. Interface of Java-based Application for Calculating the ARL

10: **End If**
11: **Loop** 4–10 for 250.000 times
12: Calculate the ARL
13: **If** the ARL does not fall within the desired value **then**
14: Change the value of L_X
15: **Repeat** 2–12
16: **Else**
17: Confirm the appropriate parameters
18: **End If**

Algorithm mEWMA Chart Simulation

1: Determine the values of ϕ, θ, λ, and L_H
2: Calculate σ_X using the square root of (2)
3: Calculate σ_H
4: Calculate the control limits using (5)
5: Generate random number $a_t \sim \text{NID}(0,1^2)$
6: Calculate X_t using (1)
7: Calculate H_t using (3)
8: **If** H_t falls within the control limits **then**
9: **Repeat** 5–7
10: **Else**
11: Calculate the Run Length (RL)
12: **End If**
13: **Loop** 5–12 for 250.000 times
14: Calculate the ARL
15: **If** the ARL does not fall within the desired value **then**
16: Change the value of L_H
17: **Repeat** 2–14
18: **Else**
19: Confirm the appropriate parameters

20: **End If**

Algortihm ARMAST Chart Simulation

1: Determine the values of ϕ, θ, and L_Z
2: Determine the initial values of u and v
3: Specify the mean shift to be detected
4: Plot R_T and R_S vs. u and v
5: **If** max $R_T > 4$ **then**
6: Confirm the value of u and v to max R_T
7: **Else If** max $R_S > 3.5$ **then**
8: Check the decreasing rate of R_T
9: **If** the rate is high **then**
10: Confirm the value of u and v so that $R_S \in [2.5, 3.5]$
11: **Else** confirm the value of u and v to max R_S
12: **End If**
13: **Else**
14: Confirm the value of u and v
15: **End If**
16: Calculate σ_X using the square root of (2)
17: Calculate σ_Z using (7)
18: Calculate the control limits
19: Generate random number $a_t \sim \mathrm{NID}(0,1^2)$
20: Calculate X_t using (1)
21: Calculate Z_t
22: **If** Z_t falls within the control limits **then**
23: **Repeat** 19–21
24: **Else**
25: Calculate the Run Length (RL)
26: **End If**
27: **Loop** 19–26 for 250.000 times
28: Calculate the ARL
29: **If** the ARL does not fall within the desired value **then**
30: Change the value of L_Z
31: **Repeat** 2–14
32: **Else**
33: Confirm the appropriate parameters
34: **End If**

RESULT AND DISCUSSION

The simulation results of this study are shown in Table 1. There are 24 scenarios with various parameters of ARMA(1,1) models. Five level of mean shifts are also incorporated to give a clear picture of the performance of the control charts in monitoring the mean shift when the process is autocorrelated. The fixed value of $\lambda = 0.2$ is used when dealing with the mEWMA chart; while various values of u and v are used in designing the ARMAST chart. The value of constants L_X, L_H, and L_Z are manipulated to give the desired value of ARL_0.

When the process is considered as weak autocorrelation, the mShewhart chart behaves similar with the conventional Shewhart chart. The constant of $L_X = 3$ (or approximately near to 3) is set to give the in-control ARL of 370. The rationale behind this similarity behavior is because the standard deviation of the weak autocorrelation process is somewhat alike with when there is no autocorrelation. In the presence of small mean shift, i.e. $0.5\sigma_X$ to $1\sigma_X$, the out-of-control ARLs are in the level of 140 to 150 and 40 to 50. It means that the control chart needs about 150 samples to give an out-of-control signal when the mean has shifted for 0.5 standard deviation. However, the mShewhart chart is very good at detecting the large mean shift. For example, when the mean shifted three standard deviation, only one sample is needed to trigger the out-of-control alarm.

TABLE 1. Simulation Result of In-Control ARL and Out-of-Control ARL

Mean Shifts	**a. ϕ = –0.95; θ = –0.90**			**b. ϕ = –0.95; θ = –0.45**			**c. ϕ = –0.95; θ = 0.00**		
	mShewhart (L_X = 2.998)	**mEWMA (L_H = 2.770)**	**ARMAST (u=0.9; v=0.1; L_Z = 2.898)**	**mShewhart (L_X = 2.675)**	**mEWMA (L_H = 1.443)**	**ARMAST (u=0.9; v=0.0; L_Z = 2.800)**	**mShewhart (L_X = 2.491)**	**mEWMA (L_H = 0.998)**	**ARMAST (u=0.9; v=0. (L_Z = 2.731**
0	370.284	370.484	370.313	370.648	370.798	370.570	370.456	370.178	370.99
$0.5\sigma_X$	147.955	32.923	28.332	158.177	8.218	7.560	144.127	4.161	3.78
$1\sigma_X$	28.792	8.293	8.300	43.366	2.752	3.184	43.287	1.255	1.35
$2\sigma_X$	1.509	2.671	2.113	1.037	1.003	1.033	1.000	1.000	1.00
$3\sigma_X$	1.004	1.193	1.0413	1.000	1.000	1.000	1.000	1.000	1.00

Mean Shifts	**d. ϕ = –0.95; θ = 0.45**			**e. ϕ = –0.95; θ = 0.90**			**f. ϕ = –0.40; θ = –0.90**		
	mShewhart (L_X = 2.434)	**mEWMA (L_H = 0.872)**	**ARMAST (u=0.9; v=0.0; L_Z = 2.589)**	**mShewhart (L_X = 2.422)**	**mEWMA (L_H = 0.837)**	**ARMAST (u=0.9; v=0.0; L_Z = 2.490)**	**mShewhart (L_X = 2.994)**	**mEWMA (L_H = 3.285)**	**ARMAST (u=0.9; v=0. L_Z = 2.838**
0	370.800	370.608	369.508	370.734	369.754	370.961	369.852	370.451	370.27
$0.5\sigma_X$	138.243	3.374	3.066	135.879	3.228	2.9564	158.146	49.660	40.19
$1\sigma_X$	42.485	1.011	1.008	42.684	1.000	1.000	45.659	12.828	12.50
$2\sigma_X$	1.000	1.000	1.000	1.000	1.000	1.000	5.367	4.068	4.42
$3\sigma_X$	1.000	1.000	1.000	1.000	1.000	1.000	1.226	2.398	2.11

Mean Shifts	**g. ϕ = –0.40; θ = –0.45**			**h. ϕ = –0.40; θ = 0.00**			**i. ϕ = –0.40; θ = 0.45**		
	mShewhart (L_X = 3.000)	**mEWMA (L_H = 2.936)**	**ARMAST (u=0.9; v=0.1; L_Z = 2.882)**	**mShewhart (L_X = 2.989)**	**mEWMA (L_H = 2.103)**	**ARMAST (u=0.9; v=0.0; L_Z = 2.791)**	**mShewhart (L_X = 2.956)**	**mEWMA (L_H = 1.409)**	**ARMAST (u=0.9; v=0. L_Z = 2.942**
0	370.687	371.099	370.885	370.473	370.779	370.457	370.485	370.344	370.72
$0.5\sigma_X$	154.458	38.423	32.308	152.565	16.995	14.573	145.082	6.679	6.17
$1\sigma_X$	42.925	10.042	10.383	42.315	5.280	5.708	41.320	2.814	1.97
$2\sigma_X$	4.880	3.434	3.666	4.456	2.118	2.622	4.297	1.273	1.41
$3\sigma_X$	1.268	1.997	1.709	1.156	1.229	1.612	1.052	1.006	1.01

Mean Shifts	**j. ϕ = –0.40; θ = 0.90**			**k. ϕ = 0.00; θ = –0.90**			**l. ϕ = 0.00; θ = –0.45**		
	mShewhart (L_X = 2.940)	**mEWMA (L_H = 1.167)**	**ARMAST (u=0.9; v=0.1; L_Z = 2.953)**	**mShewhart (L_X = 2.981)**	**mEWMA (L_H = 3.722)**	**ARMAST (u=0.9; v=0.1; L_Z = 2.773)**	**mShewhart (L_X = 2.992)**	**mEWMA (L_H = 3.533)**	**ARMAST (u=0.9; v=0. L_Z = 2.804**
0	370.994	370.126	369.612	370.104	370.323	370.533	370.386	370.163	370.71
$0.5\sigma_X$	140.348	4.657	3.895	163.049	67.084	52.610	160.526	59.638	47.63
$1\sigma_X$	40.700	2.211	2.045	50.336	17.559	16.081	47.634	15.566	14.63
$2\sigma_X$	4.325	1.069	1.054	8.441	5.313	5.667	7.522	4.893	5.26
$3\sigma_X$	1.015	1.000	1.000	2.444	3.092	3.216	2.276	2.898	2.99

Mean Shifts	**m. ϕ = 0.00; θ = 0.45**			**n. ϕ = 0.00; θ = 0.90**			**o. ϕ = 0.40 ; θ = –0.90**		
	mShewhart (L_X = 2.992)	**mEWMA (L_H = 1.880)**	**ARMAST (u=0.9; v= 0.0; L_Z = 2.867)**	**mShewhart (L_X = 2.981)**	**mEWMA (L_H = 1.356)**	**ARMAST (u=0.8; v=0.0; L_Z = 2.999)**	**mShewhart (L_X = 2.940)**	**mEWMA (L_H = 4.387)**	**ARMAST (u=0.9; v=0. L_Z = 2.769**
0	370.959	371.293	370.809	370.789	370.832	370.543	370.283	370.184	370.96
$0.5\sigma_X$	153.138	12.449	11.026	149.707	6.005	6.002	173.812	98.288	82.52
$1\sigma_X$	42.488	4.577	4.887	41.852	2.957	2.957	57.143	27.940	25.14
$2\sigma_X$	5.655	2.219	2.503	5.693	1.657	1.658	11.244	8.033	8.14
$3\sigma_X$	1.760	1.568	1.822	1.639	1.158	1.161	4.335	4.621	4.45

Mean Shifts	**p. ϕ = 0.40; θ = –0.45**			**q. ϕ = 0.40; θ = 0.00**			**r. ϕ = 0.40; θ = 0.45**		
	mShewhart (L_X = 2.956)	**mEWMA (L_H = 4.289)**	**ARMAST (u=0.9; v=0.2; L_Z = 2.791)**	**mShewhart (L_X = 2.989)**	**mEWMA (L_H = 3.834)**	**ARMAST (u=0.9; v=0.2; L_Z = 2.866)**	**mShewhart (L_X = 3.000)**	**mEWMA (L_H = 2.708)**	**ARMAST (u=0.9; v=0. L_Z = 2.910**
0	370.773	370.456	370.515	370.572	370.728	369.484	370.829	370.297	369.64
$0.5\sigma_X$	171.128	93.417	79.438	163.603	74.112	64.259	155.665	32.008	28.14
$1\sigma_X$	55.628	26.341	24.059	50.579	20.183	19.515	44.718	9.456	10.02
$2\sigma_X$	10.896	7.703	7.848	9.452	6.410	6.705	7.538	4.051	4.42
$3\sigma_X$	4.179	4.492	4.351	3.741	3.933	3.825	3.163	2.820	2.90

Mean Shifts	**s. ϕ = 0.40; θ = 0.90**			**t. ϕ = 0.95; θ = –0.90**			**u. ϕ = 0.95; θ = –0.45**		
	mShewhart (L_X = 2.994)	**mEWMA (L_H = 1.665)**	**ARMAST (u=0.9; v=0; L_Z = 2.988)**	**mShewhart (L_X = 2.422)**	**mEWMA (L_H = 5.940)**	**ARMAST (u = 0.8; v = 0; L_Z = 2.170)**	**mShewhart (L_X = 2.434)**	**mEWMA (L_H = 5.940)**	**ARMAST (u = 0.9; v = (L_Z = 2.026**
0	370.700	370.427	369.604	370.951	370.955	370.974	369.929	370.488	370.70
$0.5\sigma_X$	152.535	9.094	7..451	256.674	252.097	254.029	256.409	252.353	246.49
$1\sigma_X$	42.981	4.480	4.208	136.155	132.048	133.326	135.447	131.660	127.78
$2\sigma_X$	6.843	2.580	2.556	51.212	50.818	51.020	51.168	50.864	50.55
$3\sigma_X$	2.975	1.960	1.973	28.193	28.950	28.975	28.136	28.877	30.054

TABLE 1. (Continued)

Mean Shifts	v. ϕ = 0.95 ; θ = 0.00			w. ϕ = 0.95 ; θ = 0.45			x. ϕ = 0.95 ; θ = 0.90		
	mShewhart (L_X = 2.491)	mEWMA (L_H = 5.908)	ARMAST (u=0.9; v=0.9; L_Z = 2.491)	mShewhart (L_X = 2.675)	mEWMA (L_H = 5.686)	ARMAST (u=0.9; v=0.1; L_Z = 2.131)	mShewhart (L_X = 2.998)	mEWMA (L_H = 3.331)	ARMAST (u=0.9; v=0. L_Z = 2.490
0	370.627	370.026	370.548	370.935	370.621	370.879	371.583	370.162	370.40
$0.5\sigma_X$	255.862	250.189	254.624	247.072	239.019	234.173	179.933	89.645	80.16
$1\sigma_X$	134.394	130.338	134.213	126.655	119.871	116.123	73.374	37.038	35.24
$2\sigma_X$	50.593	50.085	50.788	47.550	45.921	45.612	28.954	18.262	18.72
$3\sigma_X$	28.027	28.462	28.028	26.736	26.508	27.385	18.486	13.020	13.80

On the other hand, in the presence of strong positive autocorrelation, the constants L_X have to be loosened to give the ARL_0 of 370. This happens because the process standard deviation is very large compared with when the process has no serial correlation. (When ϕ = 0.95 and θ = –0.9, the standard deviation is 6.0085.) A small mean shift of 0.5 standard deviation gives the value of ARL_1 more than 200. It is very undesired since the chart has already triggered an out-of-control alarm after 200 samples! The situation is opposed from the weak autocorrelation in the presence of large mean shift. Although Shewhart chart is capable of detecting large mean shift in the process, yet in strong positive autocorrelation, this capability is considered not appropriate anymore. For example, when three standard deviation of mean shift occurred, the control chart can detect the out-of-control condition after more than 25 samples!

For the mEWMA chart, as has been said before, the parameter λ is fixed at the value of 0.2 to see the effect of manipulating the constant L_H on the performance of the chart in the presence of autocorrelation. It is clear that the chart has better ability to detect small mean shift than mShewhart chart. For example, in the strong negative autocorrelation and moving average (ϕ = –0.95; θ = –0.9), the out-control ARL for detecting mean shift of $0.5\sigma_X$ is 32.9. The value is smaller when it is compared with mShewhart. (The ARL_1 is 147.96.) Although the value of ARL_1 is still smaller compared with mShewhart, the difference is slightly not huge when the process is considered as strong positive autocorrelation; say when ϕ = 0.95 and θ = 0.0, the out-of-control ARLs of mEWMA for $0.5\sigma_X$ and $1\sigma_X$ are 250 and 130 respectively (the ARL_1 of mShewhart for the same scenarios are 255 and 134). Oppositely, for examining large mean shift, the mShewhart chart is considered better than the mEWMA chart, especially for the weak autocorrelation.

Designing the ARMAST chart is tricky, since it has more parameters to be manipulated. When the optimal parameters have been found, the ARMAST chart performs better than other charts being investigated. In the case of weak autocorrelation, the out-of-control ARLs for detecting small mean shift are smaller than any other charts, say when ϕ = 0.40 and θ = 0.0, the ARL_1 for mean shift of $0.5\sigma_X$ is 64, smaller than mEWMA chart of 74 and mShewhart of 163. In the case of large mean shift, the chart is comparable with others, say for the same scenario, the ARL_1 for mean shift of $3\sigma_X$ is 3.8, while mShewhart is 3.7 and mEWMA is 3.9.

CONCLUSION AND FUTURE RESEARCH DIRECTION

This paper investigated the performance of three control charts, i.e. mShewhart, mEWMA, and ARMAST charts, in the presence of autocorrelation in the data. The simulation result which is shown in Table 1 shows that the mShewhart chart performs well when the large mean shift is occurred, but worst in detecting the small mean shift. On the other hand, mEWMA chart has a better ability in detecting small mean shift than mShewhart chart but worse in large mean shift. Among others, the well-designed ARMAST chart will be best performed, both in large and small mean shift. However, finding optimal parameters of ARMAST chart is somewhat difficult. The heuristic approach by [15] for obtaining the optimal parameters of ARMAST chart should be studied further.

The computational aspect in estimating the ARL of the control charts via simulation also should be examined. The simulation cannot give the precise value of ARL; also it spends plenty of time in obtaining the estimated ARL. The Markov chain approach, however, could be used in estimating the ARL. In addition to the Markov chain approach, the integral equation method is also a common method used to approximate the ARL [30]. However, the later cannot be used with certain kinds of control problems. For example, it cannot be used to calculate the ARL and the distribution of run-lengths [31], and therefore, the Markov chain approach gains its popularity. It has been applied in estimating the Markovian-based charting statistics, such as CUSUM [32] and EWMA [23]. Extending Markov chain approach in estimating the ARL when the IID assumption is violated will be an interesting area to be pursued.

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